SINGULAR INTEGRALS AND PARTIAL DIFFERENTIAL EQUATIONS

University of Helsinki, 24-27 May 2015

Finland

	Tue 24.5.
10:00 - 10:30	registration
10:30 - 11:15	**Volberg**
11:15 - 11:45	Coffee
11:45 - 12:30	**Volberg**
12:30 - 14:00	Lunch break
14:00 - 14:45	**Müller**
14:45 - 15:15	Coffee
15:15 - 16:00	**Kovač**
16:15 - 16:35	**Hästö**
16:40 - 17:00	**Lechner**

	Wed 25.5.	Thu 26.5.	Fri 27.5.
09:30 - 10:15	**Petermichl**	**Verbitsky**	**Auscher**
10:15 - 10:45	Coffee		
10:45 - 11:30	**Petermichl**	**Verbitsky**	**Auscher**
11:45 - 12:30	**Reguera**	**Astala**	**Fackler**
12:30 - 14:00	Lunch break		
14:00 - 14:45	**Zorin-Kranich**	**Monniaux**	**Bennett**
14:45 - 15:15	Coffee		
15:15 - 15:35	**Korte**	**Geiss**	**Wick**
15:40 - 16:00		**Bandara**	
16:15 - 16:35	**Cruz-Uribe**	**Saari**	
16:40 - 17:00		**Amenta**	

TUESDAY 24.6.

Frontiers of Singular Integrals, 2-5 June

Alexander Volberg
Michigan State University

10:30 - 11:15 11:45 - 12:30

Bellman function approach to end-point estimates of singular integrals

We will demonstrate a PDE approach to proving end-point estimates of singular integrals in unweighted and especially in weighted situations. The PDE in question are always Monge-Ampère equations with drift. In fact, the drift plays a decisive role, and we will try to show this.

Paul F.X. Müller
Johannes Kepler Universität Linz

14:00 - 14:45

Compensated compactness, interpolatory estimates, Riesz transforms, wavelet- and Haar projections

Vjekoslav Kovač
University of Zagreb

15:15 - 16:00

Norm-variation of smooth bilinear averages and several applications

We introduce smooth two-dimensional bilinear averages and establish a bound for their variation in the L^2 norm. Our primary application of this estimate is a quantitative result on the convergence of double ergodic averages with respect to two commuting transformations. This is joint work with Polona Durcik, Luka Rimanić, Kristina Ana Škreb, and Christoph Thiele.

Peter Hästö
University of Turku

16:15 - 16:35

Harnack's inequality in generalized Orlicz spaces

I present our recent proof of Harnack's inequality in generalized (Musielak-)Orlicz spaces. Special cases include variable exponent growth and the double phase functional of Barone, Colombo and Mingione. This is joint work with P. Harjulehto and O. Toivanen.

Richard Lechner
Johannes Kepler Universität Linz

16:40 - 17:00

Factorization of the identity through operators with large diagonal

Given a Banach space X with an unconditional basis, we consider the following question: does the identity on X factor through every bounded operator on X with large diagonal relative to the unconditional basis? We show that on the Banach space with an unconditional basis that Gowers constructed to resolve Banach's hyperplane problem, there exists an operator for which the answer to the question is negative. By contrast, for any operator on the mixed-norm Hardy spaces $H^p(H^q)$, where $1 \leq p, q < \infty$, with the bi-parameter Haar system, this problem always has a positive solution. The one-parameter H^p spaces were treated first by Andrew [*Studia Math.* 1979].

WEDNESDAY 24.6.

Frontiers of Singular Integrals, 2-5 June

Stefanie Petermichl

Université Paul Sabatier

9:30 - 10:15

News on multi-parameter commutators

We characterize boundedness of iterated commutators of multiplication by a symbol function and tensor products of Hilbert and Riesz transforms by means of a two-sided estimate. The upper estimate utilises work by Martikainen on shift decomposition of Journe operators. The lower estimate (the difficult part) uses a mix of operator theory through Hankel and Toeplitz forms and a geometric construction on products of spheres.

10:45 - 11:30

News on weighted estimates

The talk will cover an array of recent progress in weighted theory. We discuss the sharp weighted estimate for non-kernel operators obtained through sparse domination. We discuss very recent progress in the matrix A_2 conjecture through sparse operators. Last, we present a result in stochastic analysis using Bellman functions: the sharp weighted estimate of any pair of differentially subordinate cadlag martingales with values in Hilbert space.

Maria Carmen Reguera

University of Birmingham

11:45 - 12:30

Weighted theory for the Bergman projection, a theory of B_∞

The Bergman space $A_2(\mathbb{D})$ is the closed subspace of $L^2(\mathbb{D})$ consisting of analytic functions, where $\mathbb{D}$ denotes the unit disk. One considers the projection from $L^2(\mathbb{D})$ into $A_2(\mathbb{D})$, such a projection can be written as a convolution operator with a singular kernel. In this talk, we will present the recent advances on the one weight and two weight theory for the Bergman projection that resulted from combining techniques from complex analysis and the theory of singular integrals in harmonic analysis. We will pay special attention to the development of a B_∞ theory and its applications in Operator Theory. This is joint work with A. Aleman and S. Pott.

Pavel Zorin-Kranich

University of Bonn

14:00 - 14:45

Sparse domination of variational operators

We prove sharp weighted estimates for variational truncations of singular integral operators that are pointwise larger than the maximal truncations. To this end we apply Lacey's pointwise sparse domination technique to a variational refinement of the nontangential maximal function, unifying a number of its previous applications in the process. Joint work with de França Silva.

Riikka Korte

Aalto University

15:15 - 16:00

The space JN_p

In 1961, John and Nirenberg introduced the space of BMO functions and proved that every BMO function has exponentially decaying distribution function. In the same paper, they also introduced another space of functions that we call the John-Nirenberg space with exponent p and write JN_p. A function $u \in L^1_{loc}(Q_0)$ is in $JN_p(Q_0)$ if

$$\sup \sum_i |Q_i| \left(\fint_{Q_i} |u - u_{Q_i}| dx \right)^p < K^p$$

for some $K < \infty$, where supremum is taken over all collections of pairwise disjoint cubes Q_i in Q_0.

In this talk, we will discuss what is known about this function space and what kind of functions belong to it. It is known that $L^p \subset JN_p \subsetneq L^{p,\infty}$. We will provide an example of a function that is in $JN_p \setminus L^p$, even though every monotone JN_p-function is in L^p.

The Colloquium of the Department of Mathematics and Statistics of University of Helsinki:

David Cruz-Uribe

University of Alabama

16:15 - 17:00

The maximal operator, weights and extrapolation on variable Lebesgue spaces

The variable Lebesgue spaces are a generalization of the classical L^p spaces, replacing the constant exponent p with a function $p(\cdot)$. We will discuss recent work to establish a theory of weighted norm inequalities on these spaces, primarily by extending the theory of Rubio de Francia extrapolation to this setting. This requires a good understanding of the Hardy-Littlewood maximal operator. If time allows, we will also discuss work on the structure of A_p weights in the variable exponent setting, an area where a great deal of work remains to be done.

THURSDAY 24.6.

Frontiers of Singular Integrals, 2-5 June

Igor Verbitsky

University of Missouri

09:30 - 10:15

Weighted norm inequalities of $(1, q)$-type for integral operators and a sublinear version of Schur's lemma

We intend to present characterizations of weighted norm inequalities of $(1, q)$- type, where $0 < q < 1$, for integral operators with positive kernels (in particular, Green's kernels) and maximal functions. Our approach is based on a sublinear analogue of Schur's lemma. These inequalities are motivated by applications to certain nonlinear elliptic PDE and convolution equations with radial non-decreasing kernels.

10:45 - 11:30

Global estimates of solutions to nonlinear elliptic PDE and integral equations

We will give necessary and sufficient conditions for the existence of solutions, and provide sharp global pointwise estimates of solutions and supersolutions in terms of nonlinear potentials adapted to sublinear problems. We will also discuss global pointwise estimates of positive solutions for more general nonlinear elliptic PDE of the type $-\Delta u = \sigma u^q + \mu$ for all real q, where σ, μ are given functions, or measures, on a Euclidean domain or a weighted Riemannian manifold.

These talks are based on joint work with Alexander Grigor'yan, Dat Tien Cao, and Stephen Quinn.

Kari Astala

University of Helsinki

11:45 - 12:30

Non-linear Beltrami equations

Singular integrals and the Beurling transform in particular are known to be well related to different properties of solutions to linear Beltrami equations, i.e. 2-D quasiconformal and quasiregular mappings. The connections range from existence and regularity of solutions to a variety of other topics. Among most recent relations are for instance the linear A_2 bounds for the Beurling transform due to Petermichl and Volberg, generalised by many others, or e.g. topics related to the Iwaniec conjecture on the L^p-norm of the Beurling transform.

In this talk we wish to discuss what happens in the non-linear setting, and in particular develop regularity and other properties of solutions to the non-linear 2-D Beltrami equations. The talk is based on joint works with A. Clop, D. Faraco, J. Jääskeläinen and A. Koski.

Sylvie Monniaux

Université Aix-Marseille

14:00 - 14:45

First order approach to L^p estimates for the Stokes operator in Lipschitz domains

Stefan Geiss
University of Jyväskylä

15:15 - 15:35

Backward stochastic differential equations and harmonic analysis

We present results about quadratic and sub-quadratic BSDEs where methods from harmonic and real analysis are exploited, such as Muckenhoupt weights and an extension of Fefferman's inequality. The investigations lead to a general class of Besov spaces that a based on a decoupling technique on the Wiener space. The talk is joint work with Juha Ylinen (Jyväskylä) based on (an extension of):

[1] **S. Geiss and J. Ylinen**: Decoupling on the Wiener space and applications to BSDEs. arXiv:1409.5322v2 (78 pages)

Lashi Bandara
University of Gothenburg

15:40 - 16:00

Rough metrics, the Kato square root problem, and the continuity of a flow tangent to the Ricci flow

The Kato square root problem, resolved in 2002 by Auscher, Hofmann, Lacey, McIntosh and Tchamitchian, was formulated in terms of a first-order framework by Axelsson, Keith and McIntosh in 2005. This has allowed for the resolution of this problem on Riemannian manifolds under natural geometric assumptions. Recently, we have defined a class of Riemannian-like metrics called rough metrics that capture geometric invariances of the problem. These metrics have recently been applied to understanding a geometric flow tangential to the Ricci flow for compact manifolds with geometric singularities. In particular, we will demonstrate how the continuity of solutions to this flow can be obtained via homogeneous Kato square root estimates on compact manifolds.

Olli Saari
Aalto University

16:15 - 16:35

Parabolic BMO and the forward-in-time maximal operator

We consider parabolic BMO originating from the regularity theory of parabolic partial differential equations. It includes logarithms of positive solutions to certain parabolic PDEs, and its characteristic difference to the classical BMO is the possibility of arbitrary speed of growth in the time direction. We review some recent progress on the subject. We discuss forward-in-time maximal operator, its boundedness on parabolic BMO, and a weight class that ties these two objects together. Compared to the classical A_p weights and their theory, this more general context shows both similarities and differences.

Alex Amenta
Université Paris-Sud

16:40 - 17:00

A first-order approach to elliptic BVPs with complex coefficients and fractional regularity data

We consider the well-posedness of boundary value problems associated with elliptic equations $\operatorname{div} A\nabla u = 0$ with complex t-independent coefficients on the upper half-space, and with boundary data in Besov–Hardy–Sobolev (BHS) spaces. A key tool in our study is a theory of BHS spaces adapted to first-order operators which are bisectorial with bounded H^∞ functional calculus, and which satisfy certain off-diagonal estimates.

Within a range of exponents determined by properties of adapted BHS spaces, we show that well-posedness of a boundary value problem is equivalent to an associated projection being an isomorphism. As an application, for equations with real coefficients, we extend known well-posedness results for the Regularity problem with data in Hardy and Lebesgue spaces to a large range of BHS spaces.

FRIDAY 24.6.

Frontiers of Singular Integrals, 2-5 June

Pascal Auscher
Université Paris-Sud

09:30 - 10:15 10:45 - 11:30

The Kato square root estimate for parabolic operators

In this talk, we present the following new result established in collaboration with M. Egert and K. Nystr om. Let $A(x,t)$ be an $n \times n$ matrix on $\mathbb{R}^{n+1}$ with complex-valued, bounded, measurable entries, which satisfy a uniform accretiv- ity condition. Theorem: the operator $L = \partial_t - \mathrm{div}_x\, A(x,t)\nabla_x$ can be defined as a maximal-accretive operator in $L^2(\mathbb{R}^{n+1})$ and satisfies the Kato square root estimate: its square root has domain equal to that of the defining form. Note that we allow measurable t dependence on the coefficients.

Stephan Fackler
Universität Ulm

11:45 - 12:30

Operator theoretic tools for harmonic analysis and PDE

In this talk I present some known tools and methods from operator theory for questions arising in vector-valued harmonic analysis and PDE. These are in particular useful for problems involving rough coefficients and domains where the usual methods from harmonic analysis may not directly apply. In connection with these methods we ask for possible generalizations which lead to natural and easy to formulate open problems involving classical operator theoretic notions such as positivity and present recent partial results. I will show that some of these questions are closely related to classical open problems in operator theory.

Jonathan Bennett
University of Birmingham

14:00 - 14:45

Weighted norm inequalities for classes of oscillatory integrals

Brett Wick
Washington University in St. Louis

15:15 - 16:00

Commutators, factorization, BMO and the Hardy space